My Current Understanding of Sonic Velocity Limits

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Introduction

Over my career of supervising younger engineers, I have found that the subject of sonic velocity limitations is always a point of considerable confusion and disagreement. I believe that college and industry textbooks account for much of this, and I disagree with some published literature that says supersonic velocities can be attained in convergent-divergent nozzles, steam jets, wind tunnels, and rocket engine exhaust nozzles. Right or wrong, this article summarizes the talk I give to all new employees. Not all are totally convinced of my opinion, but it does help them to know the rules I expect them to adhere to in their engineering work for MPEC. So far, no one has explained to me how supersonic flow could occur in pipes, but I keep listening, and I am open to explanations. If any readers of this article have one, or know of one, please contact me.

Summary Conclusion

In my current opinion, vapor molecules cannot be pushed or pulled by other vapor molecules faster than the speed of sound. This is true in pipes, nozzles, steam jets, or any manner of piping possible. Unfortunately, this opinion requires that text books and articles about convergent-divergent nozzles, steam jets, supersonic wind tunnels, and “de Laval nozzles”, which assert supersonic flows up to 5+ Mach, must then be wrong. I respectfully submit that they are, because they all seem to make the same mathematical assumption that vapor exiting a restricted throat area instantly expands to fill the wider pipe area, so that the density (and therefore the velocity) of the vapor is constant throughout the cross-sectional area of the wider pipe. While this makes their calculations much easier, it is certainly an oversimplification. I believe that the exiting vapor remains dense and extremely fast (but sonic or subsonic) at the center of the wider pipe, and becomes much less dense (and also slower) at the outer area of the wider pipe. For the given mass and cross-sectional area, published simple calculations assuming uniform cross-sectional low densities do indeed calculate supersonic flow mathematically, but I believe the center core area of the flow is a much higher density, such that the vapor flow (both mathematically and actually) does NOT exceed sonic velocity. Accepting this makes our hydraulic pressure drop calculations in piping much easier to understand. Crane’s Technical Paper No. 410 titled “Flow of Fluids Through Valves, Fittings, and Pipes” (Crane’s) is a highly recommended resource that does not get confused about sonic velocity, and is the basis for all MPEC’s hydraulic calculations. I believe that this publication would probably have been a better choice for college teaching
than the college text book I had that totally confused me (and many others) with “supersonic” nozzles.

**Sonic Velocity Fundamentals**

On page 1-9 of Crane’s (25th printing, 1991) it is simply stated:

“The maximum velocity of a compressible fluid in pipe is limited by the velocity of propagation of a pressure wave which travels at the speed of sound in the fluid. Since pressure falls off and velocity increases as fluid proceeds downstream in pipe of uniform cross section, the maximum velocity occurs in the downstream end of the pipe. If the pressure drop is sufficiently high, the exit velocity will reach the speed of sound. Further decrease in the outlet pressure will not be felt upstream because the pressure wave can only travel at sonic velocity and the “signal” will never translate upstream. The “surplus” pressure drop obtained by lowering the outlet pressure after the maximum discharge has already been reached takes place beyond the end of the pipe. The pressure is lost in shock waves and turbulence of the jetting fluid.”

On Page 3-3 of Crane’s the calculation for the velocity of sound in a gas is given as:

\[ V_s = 68.1 \times \sqrt{k \times P' \times V} \]

Where \( V_s \) = speed of sound, ft/sec

\( k = \frac{C_p}{C_v} = \) Ratio of specific heats of the gas

\( P' \) = system absolute pressure, psia

And \( V \) = Actual specific volume of the gas, ft\(^3\)/lb

I believe the internet’s Wikipedia article on “Speed of Sound” also has a good discussion of the intermolecular action between molecules that decides the speed of sound, as follows:

“**Basic concept**

The transmission of sound can be illustrated by using a model consisting of an array of balls interconnected by springs. For real material the balls represent molecules and the springs represent the bonds between them. Sound passes through the model by compressing and expanding the springs, transmitting energy to neighboring balls, which transmit energy to their springs, and so on. The speed of sound through the model depends on the stiffness of the springs (stiffer springs transmit energy more quickly). Effects like dispersion and reflection can also be understood using this model.

In a real material, the stiffness of the springs is called the elastic modulus, and the mass corresponds to the density. All other things being equal (ceteris paribus), sound will travel more slowly in spongy materials, and faster in stiffer ones."
The fundamentals I get out of the above are as follows:

1. The speed of sound in a given gas is the absolute maximum velocity for that gas’s molecules, relative to each other. That is, a gas molecule cannot move closer to, or farther away from, another gas molecule faster than the speed of sound in that gas.

2. Since the formula says that $V_s$ is a function only of the gas $k$ value, and of its product $(P' \times V)$, then $V_s$ for any given type of ideal gas is constant, regardless of pressure, since $(P' \times V)$ is a constant, at a given temperature. It does change with temperature as both $k$ and $V$ change. For non-ideal gasses, there is also a small change with pressure, as its $V$ changes due to changes in its non-ideal compressibility factor ($Z$).

3. The minimum pressure in a pipe exit will be the pressure at which sonic velocity is reached, no matter how much lower the pressure is outside the pipe.

For these reasons, MPEC’s compressible pressure drop calculations always work from the most downstream point backwards. Given a destination pressure (after the pipe exit), the minimum pipe exit pressure (inside the pipe) at sonic velocity is first calculated. If this sonic exit pressure is below the actual destination pressure, then there is no sonic front, and the destination pressure is used as the ending pressure for the pipe, and an exit resistance ($K$) loss is included in the pipe’s fitting total $K$ loss. If, however, the calculated sonic exit pressure is higher than the destination pressure, then the exit loss of the pipe has to be calculated for a fake greater size pipe (to keep velocities below sonic) using ratioed equivalent higher $K$ losses (per Crane’s equation 3-24, $K_a=K_b*(D_a/D_b)^4$). If the actual destination plus the exit loss is greater than the sonic exit pressure, then the greater actual pressure is used for the pipe exit, and there is no sonic front. However, if the destination pressure plus the exit loss is still less than the sonic exit pressure for the actual exit pipe diameter, then the sonic pressure is used for the pipe exit, and the lost pressure is called the sonic front loss. Once the pipe’s inside exit pressure is set, the $\Delta P$ upstream in the same size pipe can be calculated using Crane’s equation 3-7 (compressible flow with large density changes) without worrying about sonic fronts, because at higher pressures it will always be subsonic. However, whenever the upstream pipe becomes smaller, sonic limitations at the smaller pipe exit must again be checked so that additional sonic losses can be accounted for.

**Vacuum Transfer Lines**

In refineries, the procedure above is probably most critical for sizing the line from vacuum heater outlets to vacuum column flash zones. If a sonic front loss occurs at the entrance to the vacuum column (in its flash zone), then the entrained liquid droplets are immediately exposed to much lower pressures. A 1 psi sonic front loss is very significant, due to the very low absolute pressures involved. In a typical deep vacuum column operating at a flash zone pressure of 0.5 psia (25.9 mm Hg absolute), instantly dropping 1 psi from a pipe exit (after its exit loss) of 1.48 psia, means that the system pressure drops by a factor of 3. This rapid depressurizing causes light molecules in the middle of the droplets to immediately change...
from liquid to vapor, having much, much more volume, especially because of the low pressure and high temperature. The result is “exploding” droplets so small that they form a “fog” that carries up with the vapor, even at low vapor flow rates that would normally not cause entrainment. Sonic fronts in the piping upstream of the column entrance can also cause these problems because the velocities are so high, the fog carries right into the column. For this reason, vacuum transfer lines are normally telescoped, increasing one size at a time to prevent sonic front losses. MPEC uses its “2PDP” (for 2-Phase Pressure Drop) program to design this sort of piping from the vacuum column backward all the way through the heater tubes.

Choked Flow

Sonic limits are often referred to as “choked flow” limits. This gives the false impression that no more flow can be achieved. That is only half right. No more flow can be achieved by lowering the downstream pressure, but more flow can be achieved by raising the upstream pressure. In fact, increasing upstream pressures in sonic limited situations increases flow even more than in subsonic situations. For example, in a subsonic orifice with an inlet pressure of 50 psia and an outlet pressure of 40 psia, raising the inlet pressure to 70 psia and holding the outlet at 40 psia would raise the $\Delta P$ by a factor of 3, but only achieve about 1.73 times more flow. (Subsonic flow increases by the square root of the $\Delta P$ ratio.) As the inlet pressure keeps increasing (holding the outlet constant) the flow keeps increasing by the square root of the $\Delta P$, but only until the inlet pressure gets to about twice the pressure of the exit. At that point, the velocity through the orifice becomes sonic, and it cannot go any faster. This would occur at about 80 psia inlet pressure (still keeping the outlet at 40 psia). Past this point, as the inlet pressure keeps being raised, more flow still occurs, even though it’s reached its velocity limit. At twice the inlet pressure (160 psia), the density of the gas is twice as much, so twice as much mass flow occurs at the same sonic velocity. Therefore, the mass flow is now increasing linearly with increasing pressure, instead of by the square root of the $\Delta P$. The $\Delta P$ for 160 – 40 psia is 120 psi, or 3 times the 40 $\Delta P$ where sonic velocity occurred (80 - 40 psia), so this $\Delta P$ would only have ratioed by the square root to 1.73 times as much flow, if it were still subsonic, instead of 2 times as much, being sonic. Note again, that sonic flow only increases linearly with increases in inlet pressure. Decreasing the outlet pressure to increase the $\Delta P$ results in no more flow at all, only greater sonic front losses, and more “fog” in vacuum columns. It is obviously very important to realize that completely different formulas are needed for sonic flow calculations than are needed for subsonic flows. It is also important to realize that sonic flow is not a mass flow limitation. This also points out that better vacuums (lower absolute pressures) in a vacuum column revamp, without larger transfer piping may result in much greater entrainments due to “fog”.

Sonic Booms

Jets that fly faster than the speed of sound in air cause sonic booms. I agree with all the literature and videos I see on this. The jet goes faster and faster as an opposite and equal reaction force to its thrust. The air molecules in front of the jet surfaces are pushed together
into high densities that then expand back apart after the jet surfaces pass. Vacuums are also left behind where the jet has passed because the air molecules cannot rush in as fast as the jet has left. Between air compression waves expanding from the leading edges of the jet and air rushing into voids at the trailing edges of the jet, a lot of noise is made, like thunder refilling the heated voids left by lightning. This boom radiates out from the jet at the speed of sound in all directions, and is heard along the ground wherever the jet passes by, until the jet slows to subsonic speeds. In all supersonic jet flights, note that the air itself never exceeds sonic velocity. One reason I feel sure that sonic flow is never exceeded in plant piping is because I’ve never heard any sonic booms.

My College Textbook Confusion

My college textbook was Principles of Fluid Mechanics by Richard A. Kenyon and was published in 1960. I got a lot of good out of it, but I have come to disagree with (or maybe be thoroughly confused by) 6 pages in it (131 to 137). These use the “continuity equation” and the “momentum equation” to calculate how gas can go through convergent-divergent nozzles and be accelerated to supersonic velocities. There is no discussion in the book that supersonic velocities are not possible, just mathematics to imply that it is. I believe the math is good, but the assumptions behind it are bad. To make the “continuity equation” math easier, they state the following assumption on page 131: “For one-dimensional, steady flow of a frictionless fluid, the flow is considered to have a uniform velocity distribution over the plane normal to the direction of flow at all points.” Then, on page 132 they repeat much the same thing, saying: “The flow is everywhere assumed to fill the streamtube or passage, and the velocity, pressure, temperature, and density are assumed to be uniform over a plane normal to the direction of flow at any point in the system.” I believe this is a misleading assumption. (And they don’t explain why there are no sonic booms in these supersonic nozzles.) Math can create “imaginary numbers”, and I believe that’s what this has done. If I go to the plant and drill a ¼ inch hole in a steam line, steam will shoot out of the hole at sonic velocity, and you will see the stream of tiny condensed water droplets shooting out perpendicular to the pipe. You can put your finger an inch away from the pipe and move it to within an inch of the hole and not feel anything different from ambient, but you had better not put your finger right in front of the hole, because it will tear your flesh off. If I then hold a 2’ long, 3”Ø pipe in front of the leak, so that the leak shoots down the inside of the pipe, I could assume that the vapor inside the pipe was all at atmospheric pressure with a constant density, and from the leaking mass rate, I would calculate that it was supersonic. But it’s obviously not the same pressure or density across the cross-section of the 3”Ø pipe. There is a dense center to the jetting leak, and its pressure is dissipating outward. In fact, if you hold a feather to the end of the pipe where the steam leak is entering, it will show air movement from the surroundings into the pipe (as in a steam jet).

Turbulent Free Jet Velocities

Perry’s Chemical Engineer’s Handbook (sixth edition) has a good section on “turbulent jets” (page 5-22), that I believe illustrates the steam leak described above. It also offers some equations for predicting the jetted velocity profile, and how much surrounding air is entrained.
into the jet stream. If I use these equations to estimate a jet with the stated typical jet cone angle of 20°, and assume saturated steam exiting at sonic velocity (1,643 ft/sec) through the ¼” hole at 100 psig, I calculate that the center line velocity is still essentially sonic 1.25” (5 hole diameters) away from the leak, but is only half that velocity halfway between the centerline and the outer edge of the cone, and only about 6% of sonic at the outer edge boundary of the jet. At a 5” distance from the hole (where the 20° cone is about 1.8” wide), the centerline velocity is greatly decreased to only 39% of sonic, while the mid-cone velocity is only 19% of sonic, and the outer cone velocity is only about 2% of sonic. (MPEC has a spreadsheet for this, available upon request.)

How Steam Vacuum Jets Work

Over 30 years ago, I had to inspect some vacuum steam jets being made by Graham Mfg. for a job we were doing. At Graham’s plant, I asked their chief engineer to explain to me how the steam in the jet was accelerated to supersonic velocities after leaving the steam nozzle, as their literature showed. He told me he had no idea. He said the jets were all made from experience and data points gained from real test models, and their manufacture was not based on any text book equations. The Graham head office had told them to explain the “theory” of how they worked in their literature to satisfy customers, so they had simply repeated what they found in text books. Perhaps they should have used the Perry’s “turbulent jet” literature, instead. I just checked their website while writing this article, and I see that they don’t actually state it has supersonic flow anymore, so maybe they would now agree with me (and maybe not).

So how then do they work? I believe the motive steam discharges through the steam nozzle into the converging inlet diffuser at the sonic velocity achieved at the steam nozzle hole. I do not believe that this steam is accelerated to supersonic velocity in the inlet diffuser, as much literature states (based on assuming constant cross-sectional velocities and densities). I’ve been around a lot of steam jets, and I’ve never heard a sonic boom. Instead, I believe the steam nozzle is carefully designed with a diverging exit to help shape the expanding steam jet into a widening cone shape that hits the narrowest part of the diffuser throat just right. The calculations discussed above from Perry’s “turbulent jet” literature seem very consistent with typical jet internal dimensions I’ve seen. Also, note that when you feel an operating steam vacuum jet diffuser body, you will feel that the wall of the converging inlet diffuser is about the same temperature as the suction. The jet gets hot at the narrow diffuser throat and remains hot through the diverging outlet diffuser. This clearly shows that the steam is shooting as a concentrated (high density) stream inside the converging inlet diffuser, and not along its sides at a constant cross-sectional density and at supersonic velocity. Despite being high density, the shooting steam consist of individual molecules with lots of space between them. The suction gas molecules diffuse in between the steam molecules and get entrained along with them to the jet exit. The steam molecules would also expand and diffuse into the suction gas, but their high velocity forward momentum overwhelms their outward expansion and diffusion forces, and they go forward, instead. Sweeping away the suction gas molecules creates a vacuum, so that other suction gas
molecules that are farther away rush in to fill the void. The steam and entrained suction gas are still at very high velocity in the narrow diffuser throat, but then slow down as they exit through the diverging outlet diffuser. At this point, I believe the cross-sectional density and velocity are becoming much more uniform, and the pressure is increased as the velocity is reduced, just as the text books and the Bernoulli equation predict. The velocity energy and momentum in the diffuser nozzle are the force keeping the higher pressure of the jet discharge from going backward to the jet suction nozzle. If the backpressure on the jet gets too high, however, it overpowers the momentum force and the jet vacuum “breaks” due to back-flowing “surges” where the discharge flows backwards to the suction. If the steam nozzle wears and the jetting steam starts hitting in the converging diffuser area, that also greatly hurts the performance. If the steam flow is reduced (some people try to throttle it), it doesn’t fill the narrow diffuser throat area, or it has insufficient mass, velocity, and momentum, and back-flowing also occurs.

Note that vacuum jets can use any motive gas; it doesn’t have to be steam. In fact, Hijet vacuum jets use liquid as their motive fluid. They do this by pulverizing the liquid into a very fine mist by very large pressure drop shearing nozzles. The fine droplets entrain the suction gas between them just like motive gas molecules do. The higher mass and momentum of the heavier fluid mix exiting the diffuser throat allows them to achieve much higher compression ratios than possible with vapor motive fluid jets, and it has nothing to do with supersonic or even sonic velocities.

Supersonic Wind Tunnels

I have been shown videos of supersonic wind tunnels used in designing supersonic jet planes. In these tunnels a convergent-divergent nozzle is used to accelerate the air, and stationary models of planes are held in place just downstream of the nozzle to be blasted by the “supersonic” air. I have visited NASA websites that explain wind tunnel theory and calculate moving air Mach numbers that are much greater than 1. I find that these websites use the same equations presented in my college text book, which assume constant velocity and density throughout the cross-sectional area of the divergent nozzle. Just as in the case of the steam jets, I believe the air is jetting out of the nozzle throat at sonic velocity, but in a centrally concentrated stream, and certainly not at a uniform density and velocity across the full cross-sectional area of the diffuser, or even the downstream pipe. At first it seems alarming to think that aeronautical engineers are designing supersonic jets thinking they are being tested at up to about 5 Mach, when they are really being tested at only 1 Mach, but there is a saving grace. Their 5 Mach number is calculated on the assumption that the air hitting the plane is atmospheric pressure. I believe it cannot exceed 1 Mach, but the central stream of air hitting the jet plane is about 5 times more dense than atmospheric. Since force and momentum are functions of the product of mass times velocity, these cancel out, and the force and momentum striking the jet plane is about the same, either way.

In the videos of these wind tunnel tests, you can see visible 45° lines of pressure disturbance waves form from the nose and wings of the jet planes when the gas reaches sonic speeds. As the speed is increased (supposedly) to supersonic speeds, you can see some of these
waves angle backwards, but you can still see faint 45° waves remain, as well. I suggest the
45° waves show that the air speed is no more than 1 Mach. If the air speed is 1 Mach, and
the disturbance radiates outward from the plane at 1 Mach, then vector math would support
the disturbance wave going outward at the same rate as it was carried along with the gas
(forward with the gas, but backward from the plane), and that would be a 45° angle. If the
wave were being carried 5 times as far backward (from the plane) with the Mach 5 velocity
gas, as it was radiating outward at 1 Mach, then the disturbance angle from the centerline
of the plane would only be the sine of 1/5 or 11.5°. In some of these tests you can see
multiple disturbance waves at various angles. I propose that the 45° angle disturbance is
the sonic wave, and the lesser angle lines are merely air eddies that don’t move away from
the plane at sonic velocity. I haven’t actually been able to personally witness any of these
wind tunnel tests, but I’ve listened closely to the videos that have sound, and I’ve yet to hear
a sonic boom.

de Laval Nozzles

Per Wikipedia, a de Laval Nozzle is a convergent-divergent nozzle named for its inventor,
Gustaf de Laval who applied it to steam turbines in 1888. It’s used for rocket engine exhaust
nozzles today. The literature is full of these things having supersonic velocities in the
diverging area of the nozzle. Once again, when I look at their mathematical equations, they
are assuming constant cross-sectional densities and velocities. They also seem to assume
the gas exiting into the atmosphere is at atmospheric pressure and the reaction is fully
completed. My same arguments apply here. I suspect de Laval’s invention was based on
practical experiments and not on supersonic theoretical equations. And, no, I don’t hear
sonic booms here either.

Subsonic Exhaust Behind Supersonic Jet Planes

One argument I’ve heard is that the jet exhaust behind a jet plane traveling at Mach 2 has
to be exiting the jet faster than Mach 2, or it wouldn’t be exiting. That’s just not true. Relative
to the jet from which it is exiting, it is going out at nearly the speed of sound. This drives the
jet forward. Once it leaves the jet nozzle exit, however, it enters the vacuous area left behind
the jet engine, slows down and mixes with the inrushing air that was never traveling fast.

Closing

I’m not a rocket scientist, or a NASA engineer, so there is much that I do not understand.
But I do feel the need to try and keep things logical, so that I can at least understand the
fundamentals. Therefore, I’d like to know where my fundamental (molecular level)
understanding is off on this subject. If you can educate me, I would much appreciate it. Until
then, I just can’t believe that putting an odd shaped piece in a vapor line can magically allow
the gas to become supersonic.